THE LINEARIZED POLARON MODEL SYSTEM IN A MAGNETIC FIELD

Nikolai N. Bogolubov (Jr.)

(RAS, Moscow, Russian Federation)

Abstract

This article is devoted to the calculation of the free energy on electron-phonon *interaction* in the external magnetic field according to *Green's* function method. It should be noticed that there were made the creation of structures with predicted features (with the use experimental technique). To sum up it should be noticed that the given result coincides with the result of Devreese and Brosens, which was got by *Feynman's* path integral method.

***Keywords*:** electron-phonon interaction, *Green's* function method, *Feynman's* path integral method.

1 Introduction

An electron, moving with its accompanying distortion of lattice, has been called a po-laron. It has an effective mass higher than the electron. The concept of polaron was introduced by Pecar, who considered the boundary adiabatical case, when electron-lattice interaction is so strong, that allows to describe all properties of permanent polaron in the polarized field. Such well-known investigators as Landau, Pecar, *Froehlich,*[2] *Feynman*[1] and Bogolubov introduced their contribution into evolution (developement) of polaron theory. The theory of polaron has an important part in statistical mechanics and quantum theory of field, since it can be considered as a simple instance of nonrelativistic quantum particle, interacting with quantum field. The important contribution into theory of polaron was the consruction of consistent adiabatical theory of perturbation by Bogolubov in 1950, where the kinetic energy of phonon field treated as a small perturbation. In 1980 Bogolubov returned to the problem of polaron, where he *devel oped* and applied the well-known method of chronological regulates or T — *productions*. This method was effective for polaron theory with long-range for all electron-phonon interaction forces as well as for detecting the highest series terms of the perturbation theory at the limit of weak interaction.

This electron lattice coupling though small in semiconductors, leads to perceivable effects. The polaron theory in the weak coupling limit employs either a second-order perturbation theory or a variational method. In the strong coupling one uses either *Feynman's* path integral or *Green's* function methods. It has been established that in low dimensional systems and quantum dots. These systems are interesting not only due to the technological importance, but also due to the polaronic shifts, which are quite appreciable and comparable to either the donor building energies or sub band *energ ies*. An analytical approach to obtain the electron energies and effective masses in the presence of electron-phonon coupling in a quasi-2D quantum well system has computed by several authors. In all these works variational methods have been followed; no external perturbation was included. However, the effect of electron-phonon interaction on the binding energy of the ground state of an isolated hydrogenic impurity in a *GaAs/GaAlAs* quantum subjected well to the external electric field was studied. Hai and the colleagues found that the screening due to the electron gas reduces the effective electron-phonon coupling in a Q1D system appreciably and the contribution of the electron-phonon *interaction* to the ground state energy of the polaron gas decreases with increasing electron density. Buonocore and the colleagues also calculated the po-laron self-energy for the electron-surface-phonon interaction showing that in this case the self-energy depends on the difference between the wire and embedding medium dielectric constants; it increases on reducing the wire size and can cross the volume confined polaron self-energy. It should be noticed, that one the significant problems in statistical mechanics is the investigation of dynamic process in system with weak interaction with thermostat. The first investigation was made by Bogolubov and Krylov. They developed the method, which helped on a first approximation to get Fokker-Plant equation. In lectures read by Bogolubov in 1974 at Rockefeller University was presented the modified version of the method developed by Krylov and Bogolubov (himself) and its connection with the theory of *Green's* double-time functions was discussed as well. At the heart of this method lies the elimination of Bose-variables from operator equations of motion by their averaging with the matrix initial state density. It was shown, that in case of weak interaction this equation is reducible to Bolzman equation in theory of polaron. The special attention is paid to investigation of non-equilibrium features of linealized theory of polaron. The main features of such system , impedance and admittance, are well estimated. Also it is shown, that equilibrium function of impulse distribution within weak interconnection can be obtained with the help of T-composition description with application of Boltzmann approximation equation.

2 Linearized polaron in a magnetic field

2.1 The free energy of polaron

We would like to consider a linearized model of polaron, which the first time was studied by Larsen[3] in 1964. This model is described by Hamiltonian, consisting of Hamiltonian of oscillator H_S , Hamiltonian of phonon field H_Σ and Hamiltonian of electron-phonon interaction $H_{S\Sigma}$.

$H = H_{S+\Sigma} = H_S + H_\Sigma + H_{S\Sigma},$

where

$H_S = \frac{\vec{p}^2}{2m} + \frac{\eta^2\vec{r}^2}{2}$, $H_\Sigma = \frac{1}{2}\sum_f (p_f p_f^* + \nu^2(f) q_f q_f^*)$;

$H_{S\Sigma} = \frac{K_0^2 r^2}{2} + \frac{i}{\sqrt{V}}\sum_f (S_f q_f \vec{f}\vec{r})$

Here $\vec{r}$; $\vec{p}$ are the position and impulse of an electron:

$$\vec{r} = \{x, y, z\}$$

As fas as magnetic field is oriented in a parallel way of z-axis:

$$\vec{B}(||z) = [\vec{\nabla}\vec{A}],$$

therefore

$$\vec{p} = \{p_x, p_y + m\omega_c x, p_z\},$$

where

$$\omega_c = \frac{eB}{mc}$$

is cyclotron frequency.

$S(f)$ and $\nu(f)$ are radially symmetric functions:

$$S(f) = S(|f|) > 0,$$

$$\nu(f) = \nu(|f|) > 0.$$

Summation is performed according to the majority of quasidiscrete magnitude:

$$f = (\frac{2\pi n_x}{L}, \frac{2\pi n_y}{L}, \frac{2\pi n_z}{L}),$$

where $L^3 = V$ is the volume of system and n_x, n_y, n_z are whole integers.

$$K_0^2 = \frac{1}{3V}\sum_f (\frac{S_f^2 f^2}{\nu_f^2})$$

Lets introduce Bose- amplitudes b_f, b_f^+ :

$q_f = (\frac{\hbar}{2\nu(f)})^{\frac{1}{2}}(b_f + b_{-f}^+)$; $p_f = i(\frac{\hbar\nu(f)}{2})^{\frac{1}{2}}(b_f^+ - b_{-f})$

Is seen to be

$$q_{-f} = q_f^*; \qquad p_{-f} = p_f^*$$

Introduced amplitudes satisfy next commutation relations:

$b_f b_{f_1}^+ - b_{f_1}^+ b_f = \delta_{f,f_1}$; $b_f b_{f_1} - b_{f_1} b_f = 0$; $b_f^+ b_{f_1}^+ - b_{f_1}^+ b_f^+ = 0.$

2.2 The free energy of polaron for monofrequent phonons

In the presence of magnetic field for a linear polaron we are studying F_{int} for monofrequent phonons with the given frequency ν_0:[5]

$$\Lambda_\infty(\Omega) = -\frac{K_0^2\Omega}{\Omega^2-\nu_0^2}. \tag{28}$$

Devreese, Brosens and Vansant investigated model problem of the polaron theory with the participation of Richard Feynman. Putting (28) to expression (27) for the free energy and integrating, they got:

$$F_{int} = -\frac{i\hbar}{4\pi}\int_{-\infty}^{\infty}\frac{d\omega}{1-e^{-\beta\hbar\Omega}}\,[ln(\Omega^2-\mu^2) - 3ln(\Omega^2-\nu_0^2) - ln(\Omega^2-\omega_c) +$$
$$+ln([\Omega^2-\mu^2]^2\Omega^2 - \omega_c^2(\Omega^2-\nu_0^2)^2)]_{\omega-i0}^{\omega+i0}$$

As a result, they got the following expression:

$$F_{int} = -\theta\ln\sqrt{\frac{m+\frac{K_0^2}{\nu_0}}{m}} + \frac{\hbar}{2}(\mu-\nu_0) + \frac{\hbar}{2}(\Sigma_{j=1}^3 x_j - 2\nu_0 - \omega_c) - \theta\ln[\frac{(1-e^{-\beta\hbar\nu_0})^3}{\Pi_{j=0}^3(1-e^{-\beta\hbar x_j})}] - \theta ln(1-e^{-\beta\hbar\omega_c}).$$

3 Linearized polaron in the absence of a magnetic field

For the linearized polaron model in the absence of a magnetic field we find:

$$F_{int}^{\omega_c\to 0} = -\frac{3i\hbar}{2\pi}\int_0^1 d\lambda\int_{-\infty}^{\infty}\frac{\Omega}{1-e^{-\beta\hbar\Omega}}\frac{1}{\Omega}\frac{\lambda\Lambda_\infty(\Omega)}{[m\Omega+\lambda^2\Lambda_\infty(\Omega)]}\Big|_{\omega-i0}^{\omega+i0} d\omega. \tag{29}$$

It is not difficult to integrate

$$\frac{3}{2}\theta\int_0^1 d\lambda\frac{d}{d\lambda}[ln(1-e^{-\beta\hbar\mu(\lambda)}) + ln(e^{\beta\hbar\mu(\lambda)}-1)] = \frac{3}{2}\theta(ln\frac{e^{\beta\hbar\mu}-1}{e^{\beta\hbar\nu_0}-1} + ln\frac{e^{\beta\hbar\mu}-1}{e^{\beta\hbar\nu_0}-1}) =$$
$$= \frac{3}{2}\theta(ln\frac{1-e^{-\beta\hbar\mu}}{1-e^{-\beta\hbar\nu_0}} + ln\frac{e^{\beta\hbar\mu}e^{-\beta\hbar\nu_0}(1-e^{-\beta\hbar\mu})}{1-e^{-\beta\hbar\nu_0}}) = -3\theta ln\frac{1-e^{-\beta\hbar\nu_0}}{1-e^{-\beta\hbar\mu}} + \frac{3\hbar}{2}(\mu-\nu_0).$$

They obtained:

$$F_{int}^{\omega_c\to 0} = -3\theta\ln(\frac{m+\frac{K_0^2}{\nu_0{}^2}}{m})^{\frac{1}{2}} + \frac{3\hbar}{2}(\mu-\nu_0) - 3\theta\ln\frac{1-e^{-\beta\hbar\nu_0}}{1-e^{-\beta\hbar\mu}}$$

5 Comparison of the two theories

The free energy of an harmonic oscillator in a magnetic field is:

$$F_S = \frac{3}{2}\hbar\omega_c + 3\ln(1-e^{-\beta\omega_c}). \tag{32}$$

This free energy coincides with the free energy of the particle in our linearized polaron. Landau solving Schroedinger equation for temprature $T = 0$ with Hamilton operator, contained magnetic component, obtained well-known in quantum mechanics result for energy of basic condition of electron.

$$E_n = (n+\frac{1}{2})\hbar\omega_c + \frac{h^2k_z^2}{2m},$$

where

$$\frac{m\omega_c V}{2\pi\hbar}$$

is a total degeneracy of n.

originating from the free particle $Hamiltonian$,[8] we obtain in both cases for F_{int} :

$$F_{int} = -\theta \ln \sqrt{\frac{m+\frac{K_0^2}{\nu_0}}{m}} + \frac{\hbar}{2}(\mu - \nu_0) + \frac{\hbar}{2}(\Sigma_{j=1}^3 x_j - 2\nu_0) - \theta \ln[\frac{(1-e^{-\beta\hbar\nu_0})^3}{\Pi_{j=0}^3 (1-e^{-\beta\hbar x_j})}]$$

This means we have proved that both theories obtain the same results for the free energy.

Conclusion

We have studied the linear model of polaron in the presence of magnetic field by the generalized method of Greens function, calculating the free energy of electron-phonon interaction in magnetic field. Then, turning magnetic component to "zero" we have achieved the results for free energy electron-phonon interaction in magnetic field absence, which is presented *in*.[5] We have noticed that there are approaches, based in Feyman *continual* method, in particular, the Devreese and Brosens diagonalization method. Having compared both theories, we have assured their equivalence.

References

1. R. P.Feynman, Phys. Rev. 97, 660 665 (1955)
2. Froehlich H., Pelzer H., Zineau S., Philos. Mag. 41, 221 (1950)
3. D. M.Larsen Phys Rev., p. 135 419, 1964.
4. N. N. Bogolubov and N. N. Bogolubov Jr., World Scientific (2004)
5. N. N. Bogolubov and N. N. Bogolubov Jr., Comm. JINR, 17-81-65 (1981)
6. J. T. Devreese and F. Brosens Proceeding of the Adriatico Research Conference on Path Integrals and their applications, Trieste, 1987.
7. F. M. Peeters and J. T. Devreese Phys. Rev. B. 25, 12, p. 7281 7301, 1982.
8. J. T. Devreese, F. Brosens and R. Evrard, Proceeding of the Fifth International Symposium on Selected Topics of Statistical Mecanics, Dubna (1989)